Mathematical Models of Systems

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Abstract: Classification of control object models. There is a wide variety of model types and classes. None of the classification methods gives a complete picture and does not reflect all the properties of the models used, because it characterizes only individual features of the model.

Keywords: Classification, mathematical, model, operator, object, variable, static and dynamic, linear and nonlinear, discrete and continuous, stationary and non-stationary.

Let's consider the main types of models, which are divided according to the main system features:

➢ physical (full-scale) and mathematical (symbolic);

➢ one-dimensional and multidimensional;

➢ static and dynamic;

➢ deterministic and stochastic;

➢ linear and nonlinear;

➢ discrete and continuous;

➢ stationary and non-stationary;

➢ concentrated and distributed;

➢ input-output characteristics and description in the state space;

➢ structured and aggregated;

➢ parametric and nonparametric.

Physical models are models in which the properties of a real object are represented by the characteristics of a real object of the same or similar nature. Mathematical models include those in which mathematical constructions are used to describe the characteristics of an object. In the future, we will consider only mathematical models.

One-dimensional objects are called having one input and one output, multidimensional (multi-connected) objects have several inputs and several outputs.

An object is called dynamic if its output effect depends not only on the input effect at the current time, but also on the previous input values. This means that the object has inertia (memory). Mathematical models of dynamic objects define its behavior in time.

An object is called static if its response to an input action does not depend on the background, on the behavior of the system in the past, as well as on previous input values. Static systems have an instant reaction to the input impact. Static models describe processes that do not change over time, i.e. the behavior of an object in steady-state modes.

An object is called deterministic if its output effect is uniquely determined by the structure of the

object and the input it does not depend on uncontrolled random factors. In real conditions, the observed output signals change not only under the influence of the observed inputs, but also due to numerous unobservable random interference. If these interferences are small or absent, then the system can be considered deterministic. A system in which random interference has a significant effect on output variables is called stochastic. The stochastic (probabilistic) model reflects the impact of random factors, therefore, there is not an unambiguous functional dependence between the input and output variables, but a probable one. Usually, the state variables of a stochastic object are evaluated in terms of mathematical expectation, and the input effects are determined by probabilistic distribution laws.

An object is called linear if the principle of superposition is valid for it, i.e. the reaction of an object to a linear combination (superposition) of two input influences is equal to the same combination of reactions of this object to each of the influences:

$$f\big(\alpha u_1(t) + \beta u_2(t)\big) = \alpha f\big(u_1(t)\big) + \beta f\big(u_2(t)\big), \quad (1)$$

where u1(t) and u2(t) are input effects; α and β are arbitrary coefficients. Otherwise, the object is considered non-linear.

An object is called continuous if the states of its input and output effects change or are measured continuously for a certain period of time. An object is called discrete if the state of its outputs and inputs is determined only at discrete points in time. Lattice functions, which are analogs of continuous functions, and difference equations, which are analogs of differential equations, are used to describe discrete systems.

An object is called stationary if its reaction to the same input effects does not depend on the time of application of these effects, i.e. the parameters of such an object do not depend on time. Otherwise, they say that the object is unstable.

An object is called an object with concentrated parameters if its input and output values depend only on time (only on one variable). Models of objects with concentrated parameters contain one or more time derivatives of state variables and represent ordinary differential equations. The mathematical model of transitional processes in the object, along with the differential equation, also contains additional conditions of unambiguity - initial conditions.

An object is called an object with distributed parameters if the output value depends on several variables - on time and on spatial coordinates. This situation usually occurs when the studied characteristic of the object, for example, temperature, concentration of the substance, etc., is distributed in some volume. In this case, the mathematical model of the object contains partial derivatives and describes both the dynamics of the process in time and the distribution of the characteristic in space. The mathematical model of processes in a distributed object includes a differential equation in partial derivatives, initial conditions and boundary conditions. An example of such a model can be a wave equation, a diffusion model, or a thermal conductivity model:

$$\frac{\partial Q(x,t)}{\partial t} = a\frac{\partial^2 Q(x,t)}{\partial x^2} + f\big(x, t, u(x,t)\big) \quad (2.)$$

where Q(x,t) is the state function of a one-dimensional object with distributed parameters; $x \in [x0, x1]$; a and f are the specified coefficient and function, respectively.

The characteristics of the input - output type are certain operators that connect the behavior of the output value of an object with the input, for example, transfer, transition, weight functions.

State space models describe the dynamic behavior of a system with n degrees of freedom characterized by n coordinates, called state coordinates. Such coordinates, for example, are the values of the function and its n-1 derivatives in an arbitrary moment in time. They constitute an n-dimensional vector that completely determines the state of the system at any moment in time in the n-dimensional state space or phase space. The coordinates of the state vector, in contrast to the vectors of input and output quantities, in general, are abstract mathematical characteristics,

the physical nature of which is irrelevant. The coordinates of the state vector, as well as the structure and values of the coefficients of the equations of states depend on the choice of the basis in the phase space. A specific type of equations in the state space is given below for linear and nonlinear systems.

A structured model is a representation of a mathematical model of the entire system as a whole, as a set of relatively simpler models of individual elements and blocks of an object connected to each other by means of connections. It characterizes both physical and technical aspects of building a control system and allows you to study the processes occurring both in the entire system as a whole and in its individual elements. Thus, a structured model of a control system is a set of a number of interrelated mathematical models of individual links. In such a model, consistently excluding from consideration all internal variables that are input or output signals of internal links, it is possible to find a differential equation describing the relationship between the input and output values of the system and being, in fact, an aggregated model. The aggregated model describes the functional relationships between input and output values without taking into account the internal structure and relationships in the system.

Parametric models are described by explicitly defined analytical dependencies containing parameters to be identified. These dependences represent parametric models of finite dimension, for example differential equations of a certain order, models in the state space. The parameters are the numerical values of the quantities defined bydividing the output of the model (for example, the values of coefficients of ordinary differential equations, initial conditions, coefficients of transfer functions). Methods of parametric identification determine unknown coefficients of the equation of the object or transfer function.

Nonparametric models are reduced to describing the transformations of the input space signals into the elements of the output space. In this case, the object model is determined by the operator for converting the functions of input signals into functions of output quantities. Non-parametric models are weight functions, transfer functions, (if the number of coefficients is not specified in advance), correlation functions, spectral densities, Volterra series. For example, for a model in the form of a weight function, the relationship between input and output signals for linear objects is set using the convolution integral (Duhamel integral):

$$y(t) = \int_0^\infty w(\tau)u(t-\tau)d\tau = \int_0^\infty u(\tau)w(t-\tau)d\tau \qquad (3.)$$

where w(t) is the impulse transient (weight) function of the object, which is a nonparametric model of a linear dynamic object.

Nonparametric identification methods are used to determine the time or frequency characteristics of objects. According to the obtained characteristics, you can then determine the transfer function or the equations of the object. Parametric models can lead to large errors if the order of the model does not match the order of the object. The advantage of nonparametric models is that they do not require explicit knowledge of the order of the object. However, in this case the description is essentially infinite-dimensional.

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